

## Someday, Sir, you will tax it!

A paper prepared for the Sphex Club of Lynchburg  
January 6, 2010 2011



I was originally scheduled to speak at the first meeting of the club in September, but agreed at his request to switch dates with Dr. Tiller. I had spent much of the summer thinking about and doing research on the current state of US energy policy and that would have been the topic of choice. However, as is often the case with me, my research since September has led me in a quite different direction, one more attuned to my predilection for the history of science. Besides, it's Twelfth Night, so the holiday season is still with us and the energy story is entirely too depressing for the beginning of a new year, so it will have to wait for another time.

As I think about energy and energy policy, it becomes ever more apparent that the marvelous form of energy we call electricity must play an increasingly important role. I began to think about speaking on electricity as an energy form. But that led me to the history of the subject. I emphasize that I am not an historian of science, but merely one who from time to time dips his toes in the waters. As I look from the broadest possible perspective at the history of science, I can identify only a few scientists whose thoughts have so profoundly changed the way we think that they might be described as having "changed the world." While not a scientist by modern standards, Aristotle clearly fits this description and his explanations of natural phenomena dominated Western thought for more than 15 centuries. One could include Galileo in this group, but clearly Newton and Einstein stand out in the physical sciences, Darwin in the biological sciences, Freud in the life sciences and perhaps Marx in the social sciences. I argue that there is one other whose name deserves to be mentioned in this select group—a man whose portrait, along with that of Newton, sat on Einstein's mantel throughout his adult life—but he is perhaps the least known and least appreciated of the group. I speak of James Clerk Maxwell.

I will begin by giving you some flavor of the person and then speak briefly about his contributions to science. I am heavily indebted to the biography written by his lifelong friends Lewis Campbell and William Garnett in 1882 for most of the biographical information that follows. Unless otherwise indicated, all quotes are from that work and I will use the abbreviation C & G to reference it. James Clerk Maxwell was born in Edinburgh, Scotland, in 1831. Shortly following his birth, his family inherited the country estate of Glenlair about 12 miles from Dumfries. His father, John Clerk, adopted the surname Maxwell when he inherited the estate. Although he was landed gentry, John Clerk Maxwell was a practical man—one who

designed his own square-toed shoes for comfort. He paid little attention to fashion, but much attention to his son.

*In [John Clerk's] Diary we find him cutting out his own and his son's shirts, while planning the outbuildings which still exist at Glenlair. And he not only planned these, but made the working plans for the masons with his own hand.*

The young James was naturally curious, as the following letter, written when he was not yet three years old describes:

*He is . . . very happy. . . , and has improved much since the weather got moderate; he has great work with doors, locks, keys etc., and 'Show me how it doos' is never out of his mouth. He also investigates the hidden course of streams and bell-wires, the way the water gets from the pond through the wall and a pend or small bridge and down a drain ...*

Lewis Campbell wrote: *Before seeing this letter, I had been told by his cousin, Mrs. Blackburn, that throughout his childhood his constant question was, "What's the go o' that? What does it do?" Nor was he content with a vague answer, but would reiterate, "But what's the particular go of it?" . . . I distinctly remember his telling me, during his early manhood, that his first recollection was that of lying on the grass before his father's house, and looking at the sun, and wondering.*

James entered the Edinburgh Academy in November of 1841, at the age of ten. Prior to that he had been home schooled. He was clearly out of place in this prestigious school with his homemade country clothes and square-toed shoes. Because of his appearance, his quiet demeanor, and the fact that he was not so well prepared as his classmates, his classmates immediately tagged him with the nickname 'Dafty,' which he never seemed to resent.

According to Lewis Campbell, who became his closest friend during his time at Edinburgh, *". . . At school also he gradually made his way. He soon discovered that Latin was worth learning, and the Greek Delectus interested him, when we got so far. And there were two subjects in which he at once took the foremost place, when he had a fair chance of doing so; these were Scripture Biography and English. In arithmetic, as well as in Latin, his comparative want of readiness kept him down. On the whole he attained a measure of success which helped to secure for him a certain respect, and, however strange he sometimes seemed to his companions, he had three qualities which they could not fail to understand—agile strength of limb, imperturbable courage, and profound good nature."*

P G Tait, a noted scientist as well as lifelong friend and colleague, was one class younger than Maxwell. He recollected that. . .

*At school he was at first regarded as shy and rather dull. he made no friendships and spent his occasional holidays in reading old ballads, drawing curious diagrams and making rude mechanical models. This absorption in such pursuits, totally unintelligible to his schoolfellows, who were then totally ignorant of mathematics, procured him a not very complimentary nickname. About the middle of his school career however he surprised his companions by suddenly becoming one of the most brilliant among them, gaining prizes and sometimes the highest prizes for scholarship, mathematics, and English verse.*

At the age of 14, Maxwell wrote a mathematical paper in which he generalized the concept of an ellipse. His father, who regularly attended meetings of the Royal Society of Edinburgh, sent the paper to a member of the society, who read it and forwarded it to others. They agreed that the paper had merit and *On the description of oval curves, and those having a plurality of foci* was read to the Royal Society of Edinburgh by a member of the society in April 1846. They thought it inappropriate for a 14 year old to appear before the society.

Maxwell entered Peterhouse College at Cambridge in October of 1850 but soon moved to Trinity College where he believed that it was easier to obtain a fellowship. Again I quote Tait's article in the *Proceedings of the Royal Society of Edinburgh* (1879-80):-

*... he brought to Cambridge. . . a mass of knowledge which was really immense for so young a man, but in a state of disorder appalling to his methodical private tutor. . .the student to a great extent took his own way, and it may safely be said that no high wrangler of recent years ever entered the Senate-house more imperfectly trained to produce 'paying' work than did Clerk Maxwell. But by sheer strength of intellect, though with the very minimum of knowledge how to use it to advantage under the conditions of the Examination, he obtained the position of Second Wrangler, and was bracketed equal with the Senior Wrangler, in the higher ordeal of the Smith's Prizes.*

Another friend and colleague described Maxwell's undergraduate days:-

*... Scholars dined together at one table. This brought Maxwell into daily contact with the most intellectual set in the College, among whom were many who attained distinction in later life. These in spite of his shyness and some eccentricities recognised his exceptional powers. ... The impression of power which Maxwell produced on all he met was remarkable; it was often much more due to his personality than to what he said, for many found it difficult to follow him in his*

*quick changes from one subject to another, his lively imagination started so many hares that before he had run one down he was off on another.*

Maxwell had a good sense of humor and apparently enjoyed the occasional practical joke. At one point during his youth, he was living in what we might term a boarding house. His landlady was a gardening enthusiast and one day she brought home a new plant, which she showed to all who would look at it. That evening, Maxwell purchased a larger version of the plant and switched it with the original. The next morning his landlady was excited to see how much the plant had grown overnight and called everyone to see. Maxwell repeated the exercise nightly for the next several days and every morning, the ever more excited landlady called everyone to witness the remarkable growth. And then he reversed the process.

Maxwell obtained his fellowship and graduated with a degree in mathematics from Trinity College in 1854 and he remained at Cambridge for graduate study—tutoring until he was awarded a Fellowship by Trinity College to continue work.

However, in early 1856, Maxwell's father became ill and, wanting to spend more time with him, he applied for the post of Professor of Natural Philosophy at Marischal College in Aberdeen. Maxwell travelled to Edinburgh for the Easter vacation of 1856 to be with his father and the two went together to Glenlair. On 3 April his father died and, shortly after, Maxwell returned to Cambridge as he had planned. Before the end of April he learnt that he had been appointed to the chair at Marischal College.

In November 1856 Maxwell took up the appointment in Aberdeen. When the subject announced by St John's College Cambridge for the Adams Prize of 1857 was *The Motion of Saturn's Rings* Maxwell was immediately interested. Maxwell and Tait had thought about the problem of Saturn's rings while still pupils at the Edinburgh Academy. Maxwell's research during his first two years at Aberdeen was taken up with this topic. He showed that stability could be achieved only if the rings consisted of numerous small solid particles, an explanation eventually confirmed by the Voyager spacecraft. In a letter to Lewis Campbell, written on 28 August 1857, while he was at Glenlair, Maxwell wrote:-

*I have effected several breaches in the solid ring, and now am splash into the fluid one, amid a clash of symbols truly astounding. When I reappear it will be in the dusky ring, which is something like the siege of Sebastopol conducted from a forest of guns 100 miles one way, and 30,000 miles the other, and the shot never to stop, but go spinning away round a circle, radius 170,000 miles...*

Maxwell's essay won him the Adams Prize and Airy wrote:-

*It is one of the most remarkable applications of mathematics to physics that I have ever seen.*

Maxwell became engaged to marry Katherine Mary Dewar in February 1858 and they married in June 1859. Despite the fact that he was now married to the daughter of the Principal of Marischal College, in 1860, when Marischal College and King's College combined, Maxwell, as the junior of the department, had to seek another post. His scientific work, however, had been proceeding with great success. Professor Stokes had written to him on 7 November 1857:-

*I have just received your papers on the dynamical top, etc., and the account of experiments on the perception of colour. The latter, which I missed seeing at the time when it was published, I have just read with great interest. The results afford most remarkable and important evidence in favour of the theory of three primary colour-perceptions, a theory which you, and you alone, as far as I know, have established on an exact numerical basis.*

\* In 1860 Maxwell was appointed to the chair of Natural Philosophy at King's College in London. X He was 29 years old. The six years that Maxwell spent in this post were the years when he did his most important experimental work, although the duties of the post were more demanding than those at Aberdeen. Campbell writes:

*There were nine months of lecturing in the year, and evening lectures to artisans, etc., were recognised as a part of the Professor's duties.*

In terms that most here will understand, he lectured for 15 hours per week—the equivalent of teaching five courses—during the academic year—and throughout his university career he also lectured (free) in the evenings to members of the local workforce.

Maxwell left King's College in the spring of 1865 and returned to his Scottish estate where he continued to refine his thoughts on electricity and magnetism. He made periodic trips to Cambridge and, rather reluctantly, accepted an offer from Cambridge to be the first Cavendish Professor of Physics in 1871. He designed the Cavendish laboratory, as well as most of its instruments, and helped set it up. The Laboratory was formally opened on 16 June 1874 and has since been regarded as one of the preeminent physics research facilities in the world.

\* By late in 1878, he had become terminally ill with stomach cancer. A colleague who attended Maxwell's last lecture course at Cambridge wrote:

*During the last term in May 1879 Maxwell's health evidently began to fail, but he continued to give his lectures up to the end of the term. . . To have enjoyed even a brief personal acquaintance with Professor Maxwell and the privilege of his oral instruction was in itself a*

*liberal education, nay more, it was an inspiration, because everything he said or did carried the unmistakable mark of a genius which compelled not only the highest admiration but the greatest reverence as well.*

Maxwell returned with his wife, who was also ill, to Glenlair for the summer. His health continued to deteriorate and he suffered much pain although he remained remarkably cheerful. On 8 October 1879 he returned with his wife to Cambridge but by this time he could scarcely walk. One of the greatest scientists the world has known passed away on 5 November. He was 48 years old and at the height of his intellectual powers. Curiously, this was the year of Einstein's birth. His doctor said:-

*No man ever met death more consciously or more calmly.*

While Maxwell contributed significantly to many fields of physics, chemistry, and mathematics, I intend to speak primarily of his contributions to the field of electromagnetism. By the time the young Scot was born, some significant advances had been made in beginning to understand phenomena that had been known since prehistory. The ancients had observed what we call "static" electricity in the form of lightning and other natural phenomena, including the spark that jumps from fingertip to nearby objects. They knew that rubbing an amber object would allow it to attract such things as threads and chaffs of wheat. In fact, the Greek word for amber is *ήλεκτρον* (electron). They knew about the substance we call lodestone and its ability to attract certain metallic objects. It's possible that the Maya had harnessed the peculiar ability of lodestone to orient itself in space along the earth's north-south axis as early as 1000 BCE and it's certain that the Chinese knew how to make and use magnetic compasses by 1000 CE.

By the second half of the 18<sup>th</sup> century, scientists in Europe, along with Benjamin Franklin, had begun a systematic study of static electricity. Franklin contributed significantly to our understanding of electrostatics. Using the observation that like charges repel and opposite charges attract, these researchers showed that there were only two types of static electricity which came to be known as electric charge. Franklin named these positive and negative, to stress that most objects contain equal amounts of the two kinds of charge and thus are electrically neutral. Charging an object requires friction which either removes or adds one type of charge disturbing this balance. Had Franklin been a mathematician, experiments that he carried out would have proven that the force between charges varies as the inverse square of the distance between them. But the experimental verification of this fact fell to a French scientist named Coulomb and the mathematical form of the relationship is known as Coulomb's Law. Franklin also showed that lightning is comprised of the same type of electric charge as one finds on a charged rod. However, the famous kite-flying experiment merely confirmed what had

previously been demonstrated by a Frenchman using what we would call a lightning rod instead of a kite.

✓ By 1800 Volta had succeeded in creating the first rudimentary battery which allowed the observation of sustained flow of electric charge—what we call electric current. The Danish scientist Oersted noticed, quite by accident, that whenever he energized an electric circuit that he had constructed to observe another phenomenon, the needle of a compass that happened to be lying on the lab table deflected. The interaction seemed different from gravitational and electrostatic forces, the two forces of nature then known. The force on the compass needle did not direct it to or away from the current-carrying wire, but acted at right angles to it. Oersted wrote that "the electric conflict acts in a revolving manner." The force also depended on the direction of the current, for if the flow was reversed, then the force did too.

Oersted did not fully understand his discovery, but he observed the effect was reciprocal: a current exerts a force on a magnet, and a magnet exerts a force on a current. The phenomenon was further investigated by Ampère, who discovered that two parallel current-carrying wires exerted a force upon each other: two wires conducting currents in the same direction are attracted to each other, while wires containing currents in opposite directions are forced apart.

\* Michael Faraday was perhaps the greatest experimental physicist in history. Working in the Royal Institute of London, which had been created by the American expatriate Count Rumford, Faraday did significant research in electromagnetism as well as in chemistry. Based on observations of the forces between magnets and electric currents, he invented the first electric motor. Then he asked a research question that in many ways is characteristic of the way physics progresses. If electricity, in the form of an electric current, can create magnetism, can magnetism create electricity? Answering that question required much of his adult life, but he eventually was able to show by experiment that magnetism can create electricity and in the process, he invented the electric generator. I should also point out that the American Joseph Henry also carried out significant experiments approaching the same question from a different point of view during the same time period.

Before continuing, I need to make an aside. One of the vexing problems raised by Newton's physics was the nature of the transmission of force. Newton put forward a universal law of gravitation—masses attract other masses with a force that varies as the inverse square of the distance between them. This gravitational pull explains what we call weight and it also explains the orbit of the moon about the earth, as well as the orbit of the earth about the sun. The problem is simple—how does the moon "know" that the earth is here—a quarter of a million miles away and pulling on it as if connected by cables? Newton's explanation was called "action at a distance." The earth simply exerts this gravitational force at the point where the moon

happens to be located. This was perhaps the first of many cases in physics where the mathematics makes precise predictions, but the explanation is less than satisfying.

Faraday envisioned that the interaction between magnets and between electric charges was carried out via what he called "lines or tubes of force," crudely analogous to invisible "rubber bands" stretching through otherwise empty space and pulling or pushing on the charges or magnetic poles. Faraday further introduced the idea of the electric field. The idea is that an electric charge somehow alters the space around it, and other charges interact with the altered space, i.e., the field. The electric field is defined in terms of the electric force on a charge and force is ultimately what one measures. In the same way, one can define a magnetic field or a gravitational field to explain the interaction between magnetic poles and masses. So a mass alters the space around it and other masses interact with the altered space, removing the necessity for action at a distance.

X At an early age, Maxwell became aware of Faraday's work and one of his most important achievements was his mathematical analysis and extension of Faraday's concept of electricity and magnetic lines of force. His paper *On Faraday's lines of force* was read to the Cambridge Philosophical Society in two parts, 1855 and 1856. He showed that a few relatively simple mathematical equations could express the behavior of electric and magnetic fields and their interrelation. He was 23 years old—a year out of Trinity College.

X \* { Although he had derived his mathematical relationships through use of an elaborate mental mechanical model of the interactions between electric charges and magnetic poles, he eventually realized that the model was not necessary and began to consider only the fields themselves as the important physical entities. He essentially reduced everything that was known about electricity and magnetism to a series of equations that related the sources of these phenomena—electric charge and current—to the fields and the fields to each other. He determined that any change in an electric field would be accompanied by a corresponding change in a magnetic field. The equations that he derived contained two fundamental constants whose values could be determined by direct measurement. He realized that by combining these equations, he could derive an equation that described the propagation of the electric or the magnetic field. These derived equations were identical and further were identical to a well known "classical wave equation" that described the propagation of sound waves and other types of mechanical waves. The classical wave equation contains a term that identifies the speed of the propagation. In London, around 1862, Maxwell noted that this speed is related to the product of the two fundamental constants and thus calculated that the speed of propagation of an electromagnetic field is approximately that of the speed of light. He proposed that the phenomenon of light is therefore an electromagnetic phenomenon. Maxwell wrote:

*We can scarcely avoid the conclusion that light consists in the transverse undulations of the same medium which is the cause of electric and magnetic phenomena.*

This was a remarkable result! It was a mathematical result, only loosely tied to experiment through the forenamed constants. Prior to Maxwell, the nature of light had been substantially unknown. Experiments had shown it to be a transverse wave phenomenon. But a wave is a disturbance propagated through a medium. Since light clearly travels through empty space, what is the medium that is “waving?” Scientists had proposed that space is filled with an otherwise undetectable medium they called the “ether.” But the great speed of light implied that this medium was very rigid and so one had to explain how the planets could move through it without friction—very disturbing. But Maxwell showed that the “medium” consisted of electric and magnetic fields. Light is an electromagnetic phenomenon.

It was during this time in London that the following, perhaps apocryphal, incident occurred. Maxwell was invited to a reception held in his honor by Queen Victoria. During a conversation with Chancellor of the Exchequer William Gladstone, Gladstone supposedly said something to the effect of “This is all very well, but what good is it?” To which, Maxwell responded “Someday, sir, you will tax it.”

On the back of your handout appear the four partial differential equations, now known as Maxwell's equations, which first appeared in fully developed form in *Electricity and Magnetism* (1873). Most of this work was done by Maxwell at Glenlair during the period between holding his London post and his taking up the Cavendish chair. You may look at them as you would look at an artifact in a museum, merely as objects of curiosity. But they represent one of the great achievements of science and mathematics. Everything we know about electricity is contained within these four equations and two auxiliary equations, one of which relates the fields to forces that we can measure and one that relates electric current to electric fields.

As I began this talk, I enumerated a list of scientists whose work changed the world and you may well ask, how did James Clerk Maxwell change the world? He changed the world of physics by changing the way we understand electricity and magnetism and by providing the first viable explanation of the phenomenon we call light. He also changed the way in which mathematics interacts with physics and that interaction, in which mathematics now often leads, has profoundly affected the development of physical theories in the last century. Inconsistencies between his work and that of Newton were the inspiration for Einstein's theory of relativity.

But more importantly, Maxwell's theory of electromagnetism not only showed that light is an electromagnetic wave, but also predicted the existence of other electromagnetic waves which our eyes do not perceive. In 1871, the German physicist Heinrich Hertz demonstrated the

existence of what we now call radio waves—electromagnetic disturbances that occur with a frequency much much lower than that of light. Prior to Maxwell, invention had been largely the work of tinkerers. James Watt invented the steam pump and then the steam engine by trial and error, with literally no knowledge of the underlying science. In fact, the science of thermodynamics was largely worked out as efforts to improve the efficiencies of steam engines. By the end of the nineteenth century, Marconi had patented the wireless telegraph and the seeds of radio communication had been planted. It's difficult to imagine a tinkerer building a radio with no knowledge of electromagnetic waves.

The work I have described forms only a fragment of what he published; and when we add to this the fact that Maxwell was always ready to assist those who sought advice or instruction from him, and that he read first drafts of many works by his more intimate friends (enriching them by notes, always valuable and often of the quaintest character), we may well wonder how he found time to do so much. Maxwell's early skill in writing poetry developed itself in later years into real poetic talent. But it always had an object, and often veiled the keenest satire under an air of charming innocence.

\* Maxwell was a true "renaissance" person. The C & G biography quotes extensively from his personal correspondence—one wonders when he found time to write the lengthy letters—and the correspondence shows a man absolutely committed to his friends and family, a man who regularly returned to the liberal arts side of his academic training. He continued to read Latin and Greek; he talked about philosophy; he read Spinoza, Descartes and even Kant's Critique of Pure Reason; his letters often contain verse that he had written and they consistently reveal his deep commitment to Christianity.

I quote again from C & G *As son, friend, lover [and] husband; in science, in society, in religion; whether buried in retirement or immersed in business—he is absolutely single-hearted. This is true of his mental as well as of his emotional being, for indeed they were inseparably blended. And the fixity of his devotion both to persons and ideas was compatible with all but universal sympathies and the most fearless openness of thought.*

*That marvelous interpenetration of scientific industry, philosophic insight, poetic feeling and imagination, and overflowing honor, was closely related to the profound sincerity which, after all is said, is the truest sign alike of his genius and of his inmost nature, and is most apt to make his life instructive beyond the limits of the scientific world. . . And in his life, regarded as a whole, there is a depth of goodness which can be but faintly indicated in his biography.*

This was written in 1882 before the full impact of his accomplishments were felt. So how do 20<sup>th</sup> century physicists regard him?

Einstein said that “The special theory of relativity owes its origins to Maxwell's equations of the electromagnetic field.” And “The work of James Clerk Maxwell changed the world forever.”

Richard Feynman said that “From a long view of the history of mankind — seen from, say, ten thousand years from now — there can be little doubt that the most significant event of the 19th century will be judged as Maxwell's discovery of the laws of electrodynamics. The American Civil War will pale into provincial insignificance in comparison with this important scientific event of the same decade. “

And, finally, Carl Sagan said “Maxwell's equations have had a greater impact on human history than any ten presidents. “

## Maxwell's Equations

### Coulomb's Law

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$$

The source of a static electric field is electric charge

$$\nabla \cdot \mathbf{B} = 0$$

There exists no magnetic analogue of a single electric charge

### Faraday's Law

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

The source of a time-varying electric field is a time varying magnetic field.

### Ampère's Law

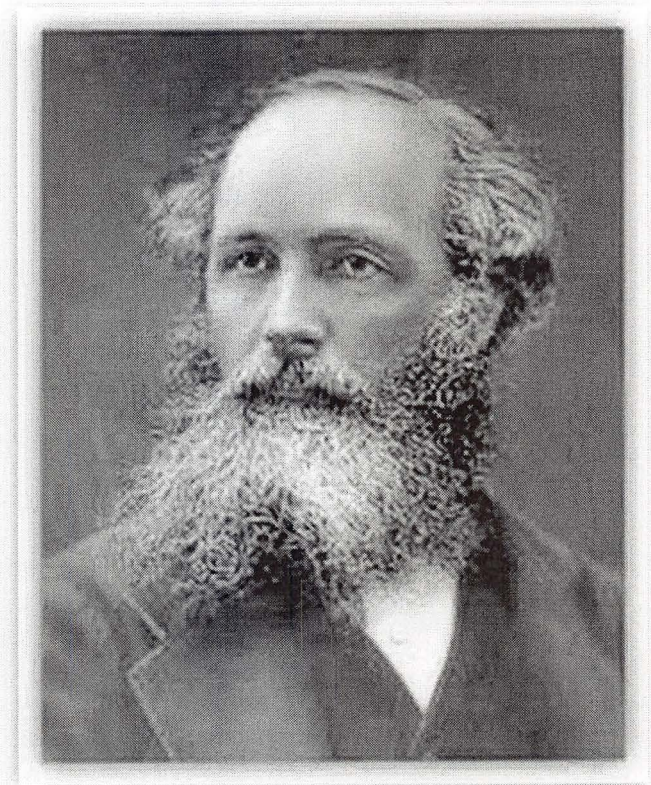
$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

The sources of a time-varying magnetic field are electric current or a time-varying electric field.

## Auxiliary Equations

**Lorentz Equation**                       $\mathbf{F} = q (\mathbf{E} + \mathbf{v} \times \mathbf{B})$

**Ohm's Law**                               $\mathbf{J} = \sigma \mathbf{E}$



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